Business Calculus - Test 2 Review
Dr. Graham-Squire, Fall 2012

1. Farmer Bob wants to enclose a rectangular area next to his barn with fencing. One side of the rectangular area will be the side of the barn, and the other three sides will be fencing. If Bob has 80 feet of fencing, his barn is 100 feet long, and the width of the rectangular area is x, find an expression for the area of the rectangle. If the width is 10 feet, what will the area be?

   Ans: $A(x) = x(80 - 2x)$ and $A(10) = 600 ft^2$.

2. Find the limits. If the limit does not exist, write DNE and explain why.

   (a) $\lim\limits_{x \to 2} \dfrac{x^2 + 4x - 12}{x^2 - 2x} = 4$

   (b) $\lim\limits_{x \to (-1)} \dfrac{x^2}{x+1}$. DNE because you are getting a number divided by zero.

   (c) $\lim\limits_{x \to \infty} \dfrac{3x^4 - 3x}{7x^2 - 11x^4 + 4} = $ -3/11

3. Let $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

   Find the value of the following limits. If the limit does not exist, write DNE and explain why.

   (a) $\lim\limits_{x \to (-1)^-} f(x) = 1$
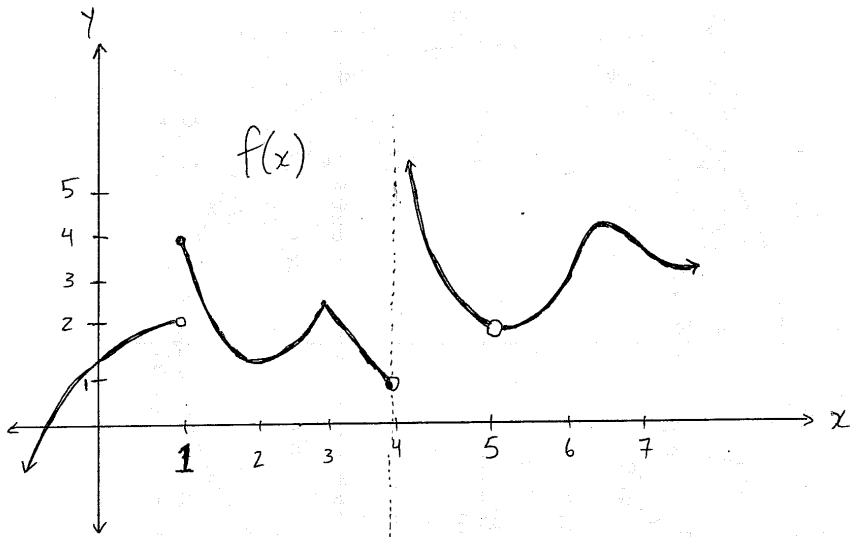
   (b) $\lim\limits_{x \to (-1)^+} f(x) = 1$

   (c) $\lim\limits_{x \to 2^-} f(x) = 4$

   (d) $\lim\limits_{x \to 2^+} f(x) = 3$

   (e) $\lim\limits_{x \to 1^-} f(x) = 1$

   (f) $\lim\limits_{x \to \infty} f(x) = 3$

4. Find the given limits for the following diagram. If the limit does not exist, write DNE and explain why.

(a) $\lim\limits_{x \to 1^+} f(x) = 4$

(b) $\lim\limits_{x \to 4^-} f(x) = 1$

(c) $\lim\limits_{x \to 4^+} f(x) = $ DNE because the graph is going to infinity.

(d) $\lim\limits_{x \to 5} f(x) = 2$

5. Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = \dfrac{1}{2x + 3}$.

   Ans: You need to do $f(x + h) =$, etc. You answer should be $f'(x) = \dfrac{-2}{(2x + 3)^2}$.

6. Find the derivative of each function:

   (a) $f(x) = (3x^4 - 7)(x^2 + 9)$. Use the product rule to get $18x^5 + 108x - 14x$

   (b) $f(x) = (x^3 - 7x + 9)^7$. Chain rule: Ans: $7(x^3 - 7x + 9)^6(3x^2 - 7)$

   (c) $f(x) = \left(\dfrac{x^3 - 9}{x + 4}\right)^3$. Ans: $\dfrac{3(x^3 - 9)^2(2x^3 + 12x^2 + 9)}{(x + 4)^4}$

   (d) $f(x) = (x + 7)^4(3x^2 - 4)^2$. Ans: $4(x + 7)^3(3x^2 - 4)(6x^2 + 21x - 4)$

7. The quantity x of TV sets demanded each week is related to the wholesale price by the equation $p = -0.006x + 180$. The weekly total cost for producing x sets is given by $C(x) = 0.00002x^3 - 0.02x^2 + 120x + 60,000$.

   (a) Find the revenue function $R(x)$ and the profit function $P(x)$.

   Ans: $R(x) = -0.006x^2 + 180x$, $P(x) = -0.00002x^3 + 0.014x^2 + 60x - 60,000$.

   (b) Compute the marginal revenue, cost, and profit functions.

   Ans: $R'(x) = -0.012x + 180$, $C'(x) = 0.00006x^2 - 0.04x + 120$, $P'(x) = -0.00006x^2 + 0.028x + 60$

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(c) Compute $R'(2000)$, $C'(2000)$, and $P'(2000)$ and interpret your results. What does that information tell the company about how many TV sets they should produce?

Ans: $R'(2000) = 156$, $C'(2000) = 280$, and $P'(2000) = -124$. At production of 2000 TV sets, the costs still exceed the revenues and the next TV made will not give any profit. Producing only 2000 TV sets is not good for profits.

8. The number of people receiving disability benefits from 1990 through 2000 is approximated by the function

$$N(t) = 0.00037t^3 - 0.0242t^2 + 0.52t + 5.3 \qquad\qquad (0 \le t \le 10)$$

where $N(t)$ is measured in units of a million and t is measured in years with $t = 0$ being 1990. Compute $N(8)$, $N'(8)$, and $N''(8)$ and interpret your results. What does that information tell you about what was happening with disability benefits at that time, and what might it imply for the future?

Ans: 8.1 million, 200,000, and 130,000. This means that in 1998, 8.1 million people were receiving disability benefits, and that number was increasing by 200,000 a year. Since the second derivative is positive, the increase is likely to continue for the next few years as well.